Discrete Mathematics - Propositional Logic

Propositional Logic

- Greek philosopher, Aristotle, was the pioneer of logical reasoning.
- Logical reasoning provides the theoretical base for many areas of mathematics and consequently computer science.
- It has many practical applications in computer science like design of computing machines, artificial intelligence, definition of data structures for programming languages etc.
- **Propositional Logic** is concerned with statements to which the truth values, “true” and “false”, can be assigned.

Prepositional Logic – Definition

- A proposition is a collection of declarative statements that has either a truth value "true" or a truth value "false".
- A proposition consists of propositional variables and connectives. We denote the propositional variables by capital letters (A, B, etc). The connectives connect the propositional variables.
- A proposition is the basic building block of logic. It is defined as a declarative sentence that is either True or False, but not both
- The **Truth Value** of a proposition is True(denoted as T) if it is a true statement, and False(denoted as F) if it is a false statement.

- examples

- "Man is Mortal", it returns truth value “TRUE”
- "12 + 9 = 3 – 2", it returns truth value “FALSE”
-
- The following is not a Proposition −

- "A is less than 2".
- What time is it?
- Go out and play.

Connectives

In propositional logic generally we use five connectives which are −

- OR (∨)
- AND (∧)
- Negation/ NOT (¬)
- Implication / if-then (→)
- If and only if (⇔).

OR (v) (disjunction)

OR ($\vee$) – The OR operation of two propositions A and B (written as $A \vee B$) is true if at least any of the propositional variable A or B is true.

The truth table is as follows –

A	B	A v B
True	True	True
True	False	True
False	True	True
False	False	False

- Two propositions ----- P , Q

- P:”today is Friday”
- Q:”It is raining today”

- Disjunction (PVQ) : today is Friday or it is raining today

AND (/\) ,Conjunction

AND ($\wedge$) – The AND operation of two propositions A and B (written as $A \wedge B$) is true if both the propositional variable A and B is true.

The truth table is as follows –

A	B	A ∧ B
True	True	True
True	False	False
False	True	False
False	False	False

- Two propositions ----- P , Q

- P:”today is Friday”
- Q:”It is raining today”
- R: 2+3=5

- Conjunction (P/\Q) : today is Friday and it is raining today
- P/\R :today is Friday and 2+3=5

Negation (~)

Negation (¬) – The negation of a proposition A (written as $\neg A$) is false when A is true and is true when A is false.

The truth table is as follows –

A	¬ A
True	False
False	True

P: “It is raining today”

- ~p: “It is not the case that is raining today”

  or simply

  “It is not raining today”.

Implication/if –then

- For any two proposition p ,q the statement " if p then q" is called implication.
- Equivalent

Implication / if-then ($\rightarrow$) – An implication $A \rightarrow B$ is the proposition "if A, then B". It is false if A is true and B is false. The rest cases are true.

The truth table is as follows –

A	B	$A \rightarrow B$
True	True	True
True	False	False
False	True	True
False	False	True

Biconditional statements

If and only if ($\Leftrightarrow$) – $A \Leftrightarrow B$ is bi-conditional logical connective which is true when p and q are same, i.e. both are false or both are true.

The truth table is as follows –

A	B	A $\Leftrightarrow$ B
True	True	True
True	False	False
False	True	False
False	False	True

References

- https://www.geeksforgeeks.org/proposition-logic/
- https://www.tutorialspoint.com/discrete_mathematics/discrete_mathematics_propositional_logic.htm#:~:text=A%20proposition%20is%20a%20collection,connectives%20connect%20the%20propositional%20variables.